SAFE FM FOR DIFFERENCE EQUATION VIBRATOR

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ABSTRACT

This paper presents a high-stability *frequency modulation* (FM) technique for a difference equation oscillator. The modulating frequency used here is an audio frequency signal, though it is possible to use also a signal below audible frequency band, or a "hidden" signal that is not connected to audio output. General operating principle as well as specific equations of the implementation are presented. External excitation has a crucial role in audio sound production.

In the beginning of the paper, is given a short introduction to classic sound synthesis techniques, and a short overview of sound analysis history in general is presented. Then a quick look is taken at some novel techniques. The meaning of noise component and variations in sound are highlighted. The operation of the proposed technique and its implementation are explained in detail.

Finally, the applications made during the experimentation are introduced: Plucked string, bell and gong. Several topologies have been evaluated. Results are reported, including notes about accuracy and limitations.

Keywords: Digital Audio Signal Processing, Sound Synthesis.

1. INTRODUCTION

We usually want to use computationally efficient methods for digital audio signal processing. But can the used method direct the results too much?

As early as in the 19th century mathematics had advanced so far that it was capable, in principle, to calculate almost any vibrations of arbitrary objects. But, in practice, scientists were interested to observe and analyse the "pure" harmonic components of the sound – and, furthermore, in a quite static way in respect to time. And, also, the "impurities" and variations as function of time were neglected as "uninteresting" [6; 7].

Still, in the 1950s, the coming computer music audio technology started by using additive synthesis (of harmonic partials). However, it became soon evident from the early experiments that the sounds thus generated did not sound very natural [4].

Quickly the so-called classic computer synthesis techniques, that included also subtractive, frequency modulation (FM) and granular synthesis – in addition to the already mentioned additive synthesis – brought variation and sense of naturalness to the resulting sound output [1].

Lately, there has been many further improvements in methods, e.g. digital waveguides [5], just to mention one. However, often there appears some kind of drawback that comes with the specialized efficiency of the method, e.g. one architecture and number of dimensions suits to model one real instrument, but another requires heavy modifications to the model to be perceived genuine enough.

Yet, authentic-sounding natural variations in sound – and the "noisy" part of the sound – have been a tough (and computationally expensive) challenge for realistic sound synthesis – in spite of the fact that, in theory, you can form any possible sound from simple sine wave components. (However, there are good analysis-resynthesis techniques [8], but they do not provide new sounds, unless you change something in the resynthesis phase to alter the characteristics[1].)

This paper will present one special method – not very efficient, but what may bring interesting results. We will first take a look into general operating principle, and then continue to the mechanism that produces variations to the sound.

2. THE DIFFERENCE EQUATION RESONATOR

The Figure 1 below shows the reference model of a basic differential equation oscillator, (similar to what

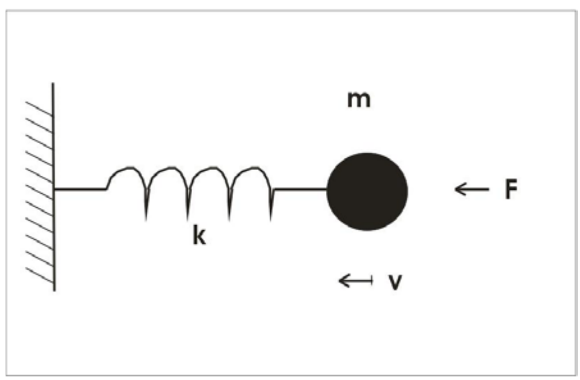

Figure 1. Basic Mass-Spring Vibrator model

you find in any elementary tutorial of mechanical vibrations, e.g. in [2; 3]), where m is mass, k is spring constant, F is force produced by the displacement and

[1] (Or, if you "fail" in the resynthesis, what may also, accidentally, produce wonderful new sounds.)

spring, and v is velocity. When we mark the displacement by y, the equation (1) below shows the physical law of one-dimensional vibration:

$$F = - k\, y \quad (1)$$

And when we mark acceleration with $\ddot{y}$ we can express the basic law of dynamics as

$$m\, \ddot{y} - F = 0. \quad (2)$$

2.1. Free vibrations

The basic reference model above showed the situation when the motion has been started and there are no resisting (or re-exciting) forces. Then the motion continues as pure harmonic (sinusoidal) motion of constant amplitude, that is called free vibration.

2.2. Forced vibrations

If there are forces affecting to the oscillating system during its vibration, the motion is not purely harmonic, and what is called forced vibration.

2.3. The Difference Equation

For the *discrete-time* version of the equation, (i.e. the implementation algorithm), we obtain[1] that the displacement force is

$$f = - k\, y[n\text{-}1] + e \quad (3)$$

where k is spring constant, $y[n\text{-}1]$ is displacement and e is external excitation force; furthermore, we get the acceleration as

$$a = f\, /\, m \quad (4)$$

where m is mass; also we get the recalculated velocity (in displacement units per sampling interval) to be

$$v[n] \;= d\; v[n\text{-}1] + a \quad (5)$$

where d is damping parameter (value range 0..1, the greater the value, the slower the decay); and, finally, we get the recalculated displacement numerical value to be

$$y[n] \;= y[n\text{-}1] + v[n] \quad (6)$$

(with the above mentioned assumption that the velocity is expressed in displacement units per unit-time, and the unit-time is set equal to one sample interval, for efficiency of calculation).

The external excitation force e is used for setting the vibrator system in motion, and it may be also used during the vibration to restrain the system from moving at its resonance frequency (or to keep the motion alive).

[1] (To save space, some steps about how we arrive to these final (optimized) equations (3), …, (6) are left out – as that is not the primary subject of this paper.)

3. FREQUENCY MODULATION (FM)

We can implement *frequency modulation* (FM) into our model by altering the spring constant k according to a modulation signal. Note that the vibration amplitude increases when the frequency decreases (a trivial law of physics for those who have ever played any string instrument). Unfortunately, that method is very prone to instability.

A safe way to implement the FM is described in Figure 2. The principle is that the difference equation calculation sequence is kept exactly same numerically also when there is FM applied. This is achieved by

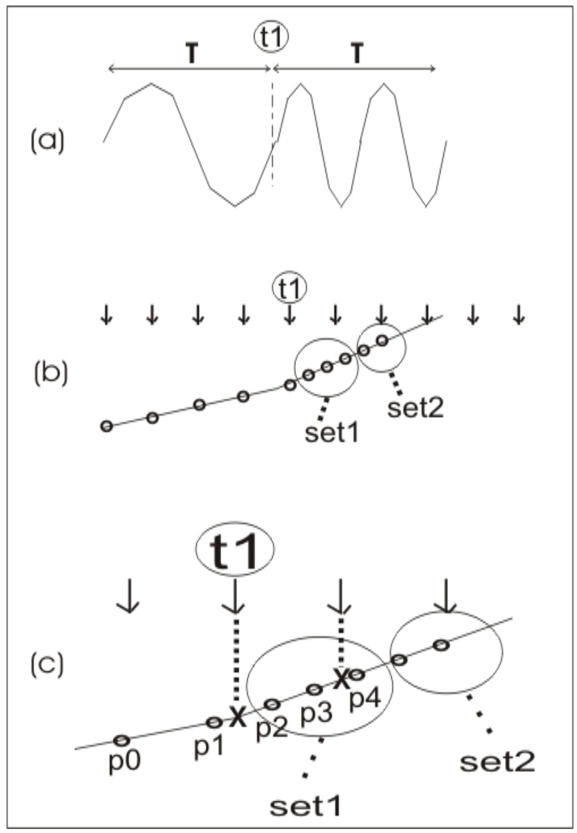

Figure 2. FM operating principle

virtually modulating the sample frequency, instead. In (a) the frequency is doubled at time *t1*, therefore in the latter time interval *T* there are double the amount of cycles than in the former interval *T*. In (b) you see the change at time *t1* in more detail: because output audio frequency was modulated to become higher, the sample frequency of calculation has to be increased. *Set1* marks the samples that have to be calculated at time *t1+1* to get a required output audio rate sample, and *set2* the samples calculated at time *t1+2*. In (c) you see that you also have to interpolate between samples to get a smooth signal for output (i.e. when the ratio of the frequencies is not a suitable integer), where *X* marks the interpolated sample for output between the difference equation sample points (*p0*, …, *p4*, etc., where we can call the line *p0*-*p1* the first "segment", and so on). Note that here, (unlike when modulating the spring constant *k*), the vibration amplitude remains naturally constant – what is actually musically desirable (because in real acoustic instruments you mostly perceive the force transferred through the bridge to the top plate of the instrument body that resonates and amplifies the sound; and changing the string stiffness does not change the vibrational energy of the string). In this safe way, implemented for the present experimentation, a "control frequency" (in Csound terminology) of equal to the audio output sample frequency was used for exchanging FM events between the resonator elements. However, the frequency of a resonator element may also be below the audible frequency range, or a "hidden" element (not connected to audio output) can be used for generating modulation control signal to other (audible) resonators. Note also, that the sample resolution decreases when FM lowers the frequency of a resonator (what must be kept in mind during the audio design).

4. APPLICATIONS AND RESULTS

For example, models resembling the following real instruments were experimented: Plucked string, bell and gong. A particular interest was to model different ways of interconnecting the resonator elements as a network, where they send and receive FM events to/from neighbour elements according to a defined topology. Several topologies of resonator networks were experimented, including linear array (plucked string physical model), and *the 120-cell* [9] (bell and gong models). The motivation for trying the 120-cell topology was to make an (artificial) resemblance to the physical construction of instruments, like the tubular bell, where there are numerous vibrational degrees of freedom, and an infinite number of closed-loop paths where the sound waves can propagate in a circular manner, thus also modifying the tension at all points of the instrument body (i.e. thus causing FM), at all times. Especially the bell and gong models gave interesting, rich timbres.

The audio sounds provided by the technique are of some interest and novelty, but not, by any means, claimed to be a general solution for synthesizing every kind of sound, nor to come without limitations. The used method can direct the results, but can it direct too much? That the author leaves for the future to show.

Below, in Figure 3, you see a curve that shows the inherent inaccuracy of the difference equation calculation with highest frequencies (i.e., with insufficient sample frequency). The linearity starts to degrade noticeably after about 5 kHz, and beyond 20 kHz the generated vibrations tend to become unstable (dashed line).

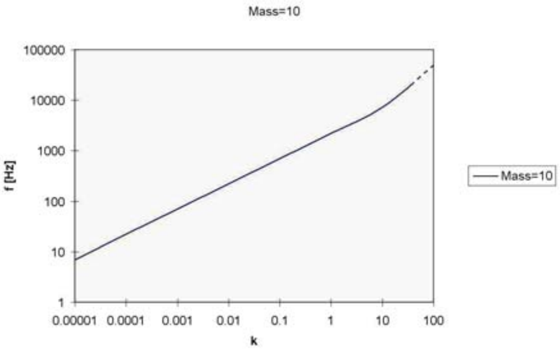

Figure 3. Linearity of difference equation calculation as the spring constant *k* and frequency *f* increase (sample frequency=44.1kHz)

The author composed his work *Fether Lyre* (2003-2004) with an environment specifically developed for this purpose, and which he named *MAL-d* (Modular Audio Laboratory (development environment)). Sound examples, and more information about the synthesis environment, including the C++ source code, can be found at:

http://www.karivakeva.com/lyre.htm

5. FURTHER WORK

One thing to study further is, for example, increasing the practical modulation depth (i.e. frequency variation range)[1].

Another thing to study is whether it could be possible to modify the current implementation so that, when you have calculated one sample segment that goes beyond subsequent output audio sample point(s) in time, you could also be able to recalculate the further end of the segment according to subsequent FM event(s), i.e. dynamic segment (sample) calculation.

Also other "closed-space" or "boundaryless" topologies could be experimented, e.g. the hypercube, the 24-cell, and the 600-cell[2].

6. REFERENCES

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[1] Note that the *resolution* of the sample frequency is not a problem, as such, because you can select the time-axis resolution of the desired signal curve representation according to the lowest audio frequency value where a resonator gets during its life-time (as a result of FM), but if the ratio between highest and lowest audio frequencies is high, you require to calculate a lot of samples at the high-end (e.g. ratio = 100 may lead to increasing the sample frequency by factor 100, which means 4.41MHz (sic) at the usual 44.1kHz output sample frequency case – *if* you want to keep a principle that sample frequency is never *less* than 44.1kHz).

[2] (A quick test with the 600-cell topology was actually made, as that topology is a *dual* of the 120-cell, and thus easy to get with the same software, and the results sounded quite similar to the 120-cell.)

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